# Corrections Savoir Fi.2

## Corrigé Exercice 6

**1)** $\int_0^1 f(t)dt = G(1) - G(0) = \left((3-1)e^1\right) - \left((0-1)e^0\right) = \mathbf{2e+1}$

**2)** **a.** $F'(x) = 2x - 2 \times \frac{1}{x} = \frac{2x^2-2}{x} = \frac{2(x^2-1)}{x} = f(x)$ donc $F' = f$ et $F$ est bien une primitive de $f$

**b.** $\int_1^e f(t)dt = F(e) - F(1) = (e^2 - 2\ln(e)) - (1 - 2\ln(1)) = e^2 - 2 - 1 + 0 = \mathbf{e^2 - 3}$

**3) a.** $F'(x) = 2ax\ln x + ax^2 \times \frac{1}{x} + 2bx = 2ax\ln x + (a+2b)x = f(x) \Rightarrow \begin{cases} 2a = 1 \\ a + 2b = 0 \end{cases} \Leftrightarrow \begin{cases} a = \frac{1}{2} \\ b = -\frac{1}{4} \end{cases}$

Donc $\boldsymbol{F(x) = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2}$

**b.** $\int_1^e f(t)dt = \left[\frac{1}{2}t^2\ln t - \frac{1}{4}t^2\right]_1^e = \left(\frac{1}{2}e^2\ln e - \frac{1}{4}e^2\right) - \left(\frac{1}{2}\times 1^2 \ln 1 - \frac{1}{4}\times 1^2\right) = \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4} = \boldsymbol{\frac{1}{4}(e^2+1)}$

## Corrigé Exercice 7

**1)**

$\mathcal{A} = \int_1^2 \left(2 - \frac{4}{x}\right) dx = [2x - 4\ln x]_1^2$
$= (2\times 2 - 4\ln 2) - (2\times 1 - 4\ln 1)$
$= 4 - 4\ln 2 - 2 + 0 = \mathbf{2 - 4\ln 2}$

$\mathcal{B} = \int_0^1 (2t^2 - 1)\, dt = \left[\frac{2}{3}t^3 - t\right]_0^1$
$= \left(\frac{2}{3}\times 1^3 - 1\right) - \left(\frac{2}{3}\times 0^3 - 0\right)$
$= \frac{2}{3} - 1 = -\mathbf{\frac{1}{3}}$

$\mathcal{C} = \int_{-1}^2 (1+u)^2\, du = \left[\frac{(1+u)^3}{3}\right]_{-1}^2$
$= \left(\frac{(1+2)^3}{3}\right) - \left(\frac{(1-1)^3}{3}\right)$
$= 9 - 0 = \mathbf{9}$

$\mathcal{D} = \int_0^3 e^{3\lambda}\, d\lambda = \left[\frac{1}{3}e^{3\lambda}\right]_0^3$
$= \left(\frac{1}{3}e^{3\times 3}\right) - \left(\frac{1}{3}e^{3\times 0}\right)$
$= \frac{1}{3}e^9 - \frac{1}{3} = \boldsymbol{\frac{1}{3}(e^9 - 1)}$

$\mathcal{E} = \int_0^{e-1} \frac{1}{x+1}\, dx = [\ln(x+1)]_0^{e-1}$
$= (\ln(e-1+1)) - (\ln(0+1))$
$= \ln e - \ln 1 = \mathbf{1}$

$\mathcal{F} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin(2x)\, dx = \left[-\frac{1}{2}\cos(2x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$
$= \left(-\frac{1}{2}\cos(2\times\frac{\pi}{2})\right) - \left(-\frac{1}{2}\cos(2\times\frac{\pi}{4})\right)$
$= -\frac{1}{2}\cos\pi + \frac{1}{2}\cos\frac{\pi}{2} = -\frac{1}{2}\times(-1) + 0 = \mathbf{\frac{1}{2}}$

$\mathcal{G} = \int_{-2}^{-1} \left(\frac{x^3}{2} - \frac{2}{x^3}\right) dx = \left[\frac{x^4}{8} + \frac{1}{x^2}\right]_{-2}^{-1}$
$= \left(\frac{(-1)^4}{8} + \frac{1}{(-1)^2}\right) - \left(\frac{(-2)^4}{8} + \frac{1}{(-2)^2}\right)$
$= \frac{1}{8} + 1 - \frac{16}{8} - \frac{1}{4} = -\mathbf{\frac{9}{8}}$

$\mathcal{H} = \int_{-1}^0 \left(1 + \frac{1}{2}e^{-t}\right) dt = \left[t - \frac{1}{2}e^{-t}\right]_{-1}^0$
$= \left(0 - \frac{1}{2}e^0\right) - \left(-1 - \frac{1}{2}e^{-(-1)}\right)$
$= -\frac{1}{2} + 1 + \frac{1}{2}e = \boldsymbol{\frac{1}{2}(1+e)}$

$\mathcal{I} = \int_1^4 \left(\frac{2}{\sqrt{x}} + x\right) dx = \left[4\sqrt{x} + \frac{x^2}{2}\right]_1^4$
$= \left(4\sqrt{4} + \frac{4^2}{2}\right) - \left(4\sqrt{1} + \frac{1^2}{2}\right)$
$= 8 + 8 - 4 - \frac{1}{2} = \mathbf{\frac{23}{2}}$

$\mathcal{J} = \int_0^1 \frac{e^{-x}}{1+e^{-x}}\, dx = [-\ln(1+e^{-x})]_0^1$
$= (-\ln(1+e^{-1})) - (-\ln(1+e^0))$
$= -\ln\left(1 + \frac{1}{e}\right) + \ln 2 = \boldsymbol{\ln\left(\frac{2e}{e+1}\right)}$

$$\mathcal{K} = \int_{-2}^{3}(t^2-4t)\,dt = \left[\frac{t^3}{3}-2t^2\right]_{-2}^{3}$$
$$= \left(\frac{3^3}{3}-2\times 3^2\right)-\left(\frac{(-2)^3}{3}-2(-2)^2\right)$$
$$= 9-18+\frac{8}{3}+8=\mathbf{\frac{5}{3}}$$

$$\mathcal{M} = \int_{1}^{e}\frac{\ln x}{x}\,dx = \int_{1}^{e}\frac{1}{x}\ln x\,dx = \left[\frac{1}{2}(\ln x)^2\right]_{1}^{e}$$
$$= \left(\frac{1}{2}(\ln e)^2\right)-\left(\frac{1}{2}(\ln 1)^2\right)$$
$$= \frac{1}{2}\times 1^2-0=\mathbf{\frac{1}{2}}$$

**2)** $I = \int_{e}^{e^2}\frac{1}{x\ln x}\,dx = [\ln(\ln x)]_{e}^{e^2} = (\ln(\ln(e^2))) - (\ln(\ln e)) = \ln(2\ln e) - \ln(1) = \mathbf{\ln 2}$

## Corrigé Exercice 8

On a $3-\frac{3}{1+e^{-2x}} = \frac{3(1+e^{-2x})-3}{1+e^{-2x}} = \frac{3e^{-2x}}{1+e^{-2x}}$

Donc

$$\int_0^a\left(3-\frac{3}{1+e^{-2x}}\right)dx = \int_0^a\frac{3e^{-2x}}{1+e^{-2x}}\,dx = \left[\frac{3}{-2}\ln(1+e^{-2x})\right]_0^a$$

$$= \left(-\frac{3}{2}\ln(1+e^{-2a})\right)-\frac{3}{2}\ln(1+e^{0})$$
$$= -\frac{3}{2}\ln(1+e^{-2a})+\frac{3}{2}\ln 2$$
$$= \mathbf{\frac{3}{2}\ln\left(\frac{2}{1+e^{-2a}}\right)}$$

**CQFD**


## Corrigé Exercice 9

1) $F_1(x) = \int_1^x(1-e^{2\lambda})\,d\lambda = \left[\lambda-\frac{1}{2}e^{2\lambda}\right]_1^x = \left(x-\frac{1}{2}e^{2x}\right)-\left(1-\frac{1}{2}e^{2}\right) = \mathbf{x-\frac{1}{2}e^{2x}-1+\frac{1}{2}e^{2}}$

$F_2(x) = \int_e^x\frac{2}{t}\,dt = [2\ln t]_e^x = (2\ln x)-(2\ln e) = \mathbf{2(\ln x-1)}$

$F_3(x) = \int_x^1(2a-3)da = [a^2-3a]_x^1 = (1^2-3\times 1)-(x^2-3x) = \mathbf{-x^2+3x-2}$

**2)** a. $a+\frac{b}{x+3} = \frac{ax+3a+b}{x+3} = \frac{1-x}{x+3} \Rightarrow \begin{cases} a=-1 \\ 3a+b=1\end{cases} \Leftrightarrow \begin{cases} a=-1 \\ b=4\end{cases}$

b. $G(T) = \int_0^T\frac{(1-x)}{x+3}dx = \int_0^T\left(-1+\frac{4}{x+3}\right)dx = [-x+4\ln(x+3)]_0^T = (-T+4\ln(T+3))-(0+4\ln(3))$

$\mathbf{G(T) = -T+4\ln\left(\frac{T+3}{3}\right)}$

c. $\int_0^1\frac{(1-x)}{x+3}dx = G(1) = -1+4\ln\left(\frac{1+3}{3}\right) = \mathbf{-1+4\ln\frac{4}{3}}$