Corrections Savoir Fd. 5

Corrigé Exercice 13

1. a) $f'(x) = 1\times e^x + x\times e^x = \boldsymbol{(1+x)e^x}$

b) $g'(x) = (3-3x^2)e^x + (3x-x^3)e^x$
$= \boldsymbol{(3+3x-3x^2-x^3)e^x}$

c) $h'(x) = 5e^x + (5x-1)e^x = (5+5x-1)e^x = (4+5x)e^x$
CQFD

d) $i'(x) = 3e^{-x} + 3x(-e^{-x}) = 3e^{-x} - 3xe^{-x} = \boldsymbol{(3-3x)e^{-x}}$

e) $j'(x) = 10e^{2-0,3x} + (10x-50)(-0,3e^{2-0,3x})$
$= (10-3x+15)e^{2-0,3x} = \boldsymbol{(-3x+25)e^{2-0,3x}}$

f) $k'(x) = 4xe^{-\frac{x}{2}} + 2x^2\left(-\frac{1}{2}e^{-\frac{x}{2}}\right) = \boldsymbol{(-x^2+4x)e^{-\frac{x}{2}}}$

2. a) $f'(x) = 3\ln x + 3x\times\frac{1}{x} = \boldsymbol{3\ln x + 3}$

b) $g'(x) = 1(2\ln x - 1) + x\left(\frac{2}{x}+0\right) = 2\ln x - 1 + \frac{2x}{x} =$
$\boldsymbol{2\ln x + 1}$

c) $h'(x) = 2x\ln x + x^2\times\frac{1}{x} = \boldsymbol{2x\ln x + x} = x(2\ln x + 1)$

d) $l'(x) = \ln(x^2+1) + x\times\frac{2x}{x^2+1} = \boldsymbol{\ln(x^2+1) + \frac{2x^2}{x^2+1}}$

e) $j'(x) = \ln(3x) + x\times\frac{3}{3x} = \boldsymbol{\ln(3x) + 1}$

f) $k'(x) = 0 + \left(0-\frac{1}{x}\right)\ln(x-1) + (1-\ln x)\times\frac{1}{x-1}$
$= \boldsymbol{\frac{1}{x}\ln(x-1) + \frac{1-\ln x}{x-1}}$

3. a) $k'(x) = 4x(\frac{x^3}{3} - 2x) + (2x^2-5)(x^2-2)$
$= \frac{4}{3}x^4 - 8x^2 + 2x^4 - 9x^2 + 10 = \boldsymbol{\frac{10}{3}x^4 - 17x^2 + 10}$

b) $h'(x) = (-1)\sqrt{x} + (2-x)\times\frac{1}{2\sqrt{x}} = -\sqrt{x} + \frac{2-x}{2\sqrt{x}} = -\frac{2\sqrt{x}\sqrt{x}}{2\sqrt{x}} +$
$\frac{2-x}{2\sqrt{x}}$
$= \frac{-2x+2-x}{2\sqrt{x}} = \boldsymbol{\frac{-3x+2}{2\sqrt{x}}}$

Pour préparer le contrôle …

1. a) $m'(x) = 3 + 2e^x + 2xe^x$
$= \boldsymbol{3 + 2(1+x)e^x}$

b) $f'(x) = 0 - (2e^x + (2x-4)e^x)$
$= -2e^x - 2xe^x + 4e^x$
$= 2e^x - 2xe^x = \boldsymbol{2(1-x)e^x}$
CQFD

c) $a'(x) = \boldsymbol{e^{-x}}$

d) $s'(x) = (-1)e^{1-x} + (1-x)(-e^{1-x})$
$= \boldsymbol{(x-2)e^{1-x}}$

2. a)
$m'(x) = 5 - \ln x - x\times\frac{1}{x} = \boldsymbol{4 - \ln x}$

b) $p'(x) = 2x\ln(2x) + x^2\times\frac{2}{2x}$
$= \boldsymbol{2x\ln(2x) + x}$
$= x(2\ln(2x) + 1)$

c) $Y'(x) = 2x\ln(x-2) + x^2\times\frac{1}{x-2}$
$= \boldsymbol{2x\ln(x-2) + \frac{x^2}{x-2}}$

3. a)
$n'(x) = -6x(x+x^3) - 3x^2(1+3x^2)$
$= -6x^2 - 6x^4 - 3x^2 - 9x^4$
$= \boldsymbol{-15x^4 - 9x^2} = -3x^2(5x^2+3)$

b) $m'(x) = 6x\sqrt{x} + \frac{3x^2}{2\sqrt{x}} = \frac{12x\left(\sqrt{x}\right)^2 + 3x^2}{2\sqrt{x}}$
$= \frac{12x^2+3x^2}{2\sqrt{x}} = \boldsymbol{\frac{15x^2}{2\sqrt{x}}}$

Corrigé Exercice 14

1. a) $j'(x) = \dfrac{e^x(2x+1) - e^x \times 2}{(2x+1)^2} = \dfrac{\mathbf{2xe^x - e^x}}{\mathbf{(2x+1)^2}} = \dfrac{(2x-1)e^x}{(2x+1)^2}$

b) $h'(x) = \dfrac{-e^x(1+e^x) - e^x(1-e^x)}{(1+e^x)^2} = \dfrac{-e^x - e^{2x} - e^x + e^{2x}}{(1+e^x)^2} = \dfrac{\mathbf{-2e^x}}{\mathbf{(1+e^x)^2}}$

c) $n'(x) = \dfrac{e^{2x} + 2 - x(2e^{2x})}{(e^{2x}+2)^2} = \dfrac{\mathbf{2 + (1-2x)e^{2x}}}{\mathbf{(e^{2x}+2)^2}}$

e) $w'(x) = \dfrac{e^x e^{3x} - (e^x - 1)(3e^{3x})}{(e^{3x})^2} = \dfrac{(e^x - 3e^x + 3)e^{3x}}{e^{6x}}$

$= \dfrac{(-2e^x + 3)e^{3x}}{e^{6x}} = \dfrac{\mathbf{-2e^x + 3}}{\mathbf{e^{3x}}}$

2. a) $j'(x) = \dfrac{\frac{1}{x} \times x^2 - 2x(\ln x + 1)}{x^4} = \dfrac{\mathbf{x - 2x(\ln x + 1)}}{\mathbf{x^4}} = \dfrac{x(1 - 2\ln x - 2)}{x^4}$

$= \dfrac{x(-2\ln x - 1)}{x^4}$

b) $k'(x) = \dfrac{x}{4\ln x} = \dfrac{4\ln x - x \times \frac{4}{x}}{16(\ln x)^2} = \dfrac{4\ln x - 4}{16(\ln x)^2} = \dfrac{\mathbf{\ln x - 1}}{\mathbf{4(\ln x)^2}}$

c) $m'(x) = \dfrac{\frac{3}{x}(1-x) - (-1) \times 3\ln x}{(1-x)^2} = \dfrac{\frac{3(1-x)}{x} + 3\ln x}{(1-x)^2} = \dfrac{\mathbf{3(1-x) + 3x\ln x}}{\mathbf{x(1-x)^2}}$

d) $f'(x) = \dfrac{\frac{2}{2x-4} \times x - 1 \times \ln(2x-4)}{x^2} = \dfrac{2}{2x-4} \times \dfrac{x}{x^2} - \dfrac{1}{x^2}\ln(2x-4)$

$= \dfrac{\mathbf{1}}{\mathbf{x^2 - 2x}} - \dfrac{\mathbf{1}}{\mathbf{x^2}}\mathbf{\ln(2x-4)}$

3. a) $f'(x) = \dfrac{6x \times (1-x) - 3x^2 \times (-1)}{(1-x)^2} = \dfrac{6x - 6x^2 + 3x^2}{(1-x)^2} = \dfrac{\mathbf{-3x(x-2)}}{\mathbf{(1-x)^2}}$

b) $g'(x) = \dfrac{2(3-4x) - (-4)(2x+1)}{(3-4x)^2} = \dfrac{\mathbf{10}}{\mathbf{(3-4x)^2}}$

c) $i(x) = \dfrac{3x^2 \times \sqrt{x} - \frac{1}{2\sqrt{x}} \times x^3}{\left(\sqrt{x}\right)^2} = \dfrac{3x^2 \times 2\sqrt{x}\sqrt{x} - x^3}{2\sqrt{x} \times x}$

$= \dfrac{6x^3 - x^3}{2x\sqrt{x}} = \dfrac{5x^3}{2x\sqrt{x}} = \dfrac{\mathbf{5x^2}}{\mathbf{2\sqrt{x}}} = \dfrac{\mathbf{5x\sqrt{x}}}{\mathbf{2}}$

Pour préparer le contrôle ...

1. a) $p'(x) = \dfrac{e^x(1+e^x) - e^x \times e^x}{(1+e^x)^2}$

$= \dfrac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2} = \dfrac{\mathbf{e^x}}{\mathbf{(1+e^x)^2}}$

b) $u'(x) = \dfrac{3e^{3x}(2x+1) - 2e^{3x}}{(2x+1)^2}$

$= \dfrac{\mathbf{(6x+1)e^{3x}}}{\mathbf{(2x+1)^2}}$

2. a)

$k'(x) = \dfrac{\left(1 + \frac{1}{x}\right)(x\ln x) - (x + \ln x)\left(\ln x + x \times \frac{1}{x}\right)}{(x\ln x)^2}$

$= \dfrac{x\ln x + \ln x - (x\ln x + x + (\ln x)^2 + \ln x)}{(x\ln x)^2}$

$= \dfrac{x\ln x + \ln x - x\ln x - x - (\ln x)^2 - \ln x}{(x\ln x)^2}$

$= \dfrac{\mathbf{-x - (\ln x)^2}}{\mathbf{(x\ln x)^2}}$

b) $g'(x) = \dfrac{\frac{2}{2x} \times (x+1) - \ln(2x)}{(x+1)^2}$

$= \dfrac{\frac{x+1}{x} - \ln(2x)}{(x+1)^2} = \dfrac{\frac{x+1-x\ln(2x)}{x}}{(x+1)^2}$

$= \dfrac{x+1-x\ln(2x)}{x\,(x+1)^2}$

3. $m'(x) = \dfrac{4x^3(x^3-1) - 3x^2(x^4+1)}{(x^3-1)^2}$

$m'(x) = \dfrac{\mathbf{x^6 - 4x^3 - 3x^2}}{\mathbf{(x^3-1)^2}}$