Corrections Savoir Sr. 2

Corrigé Exercice 2

1) $A = 0 \times n + 1 \times n + 2n + 3n = n + 2n + 3n = \mathbf{6n}$

$B = 3 \times 0^2 + 3 \times 1^2 + 3 \times 2^2 + 3 \times 3^2 + 3 \times 4^2$
$B = 3 + 12 + 27 + 48 = \mathbf{90}$

$C = 5 \times 0 + 5 \times 1 + 5 \times 2 + \cdots + 5m = \mathbf{5 + 10 + \cdots + 5m}$

$\boldsymbol{E} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \mathbf{\frac{137}{60}}$ $\quad$ $\boldsymbol{F} = 0 + \frac{2}{3} + \frac{4}{5} + \frac{6}{7} = \mathbf{\frac{244}{105}}$

$G = \frac{-1}{1} + \frac{(-1)^2}{2} + \frac{(-1)^3}{3} + \frac{(-1)^4}{4} + \frac{(-1)^5}{5} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$
$G = -\mathbf{\frac{47}{60}}$

Un peu plus…

$D = \sin(0) + \sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right) + \sin\pi$

$D = 0 + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 0 = \boldsymbol{\sqrt{3}}$

$H = 0 \times x + 1x^2 + 2x^3 + \cdots + nx^{n+1}$
$H = x^2 + 2x^3 + \cdots + nx^{n+1}$

2) $\boldsymbol{A = \sum_{k=5}^{25} k}$ $\quad$ $\boldsymbol{B = \sum_{k=5}^{30} \frac{7}{k}}$ $\quad$ $\boldsymbol{D = \sum_{k=1}^{n} 2k = \sum_{k=0}^{n} 2k}$ $\quad$ $\boldsymbol{E = \sum_{k=1}^{n} \frac{1}{k^2}}$ $\quad$ $\boldsymbol{C = \sum_{k=3}^{5} \sqrt{k}}$ $\quad$ $\boldsymbol{F = \sum_{k=1}^{n} \frac{k+1}{k}}$

Corrigé Exercice 3

$S_n = 3 + 3 \times 2^2 + \ \ldots + 3n^2$
et $S_{n+1} = 3 + \ \ldots + 3n^2 + 3(n+1)^2$
Donc $\boldsymbol{S_{n+1} = S_n + 3(n+1)^2}$

$T_n = 0 \times 1 + 1 \times 2 + \cdots + (n-1) \times (n-1+1)$
$T_n = 2 + 6 + \cdots + (n-1)n$
et $T_{n+1} = 2 + 6 + \cdots + (n-1)n + n(n+1)$
Donc $\boldsymbol{T_{n+1} = T_n + n(n+1)}$

$U_n = 0 \times x^0 + 1 \times x^1 + 2 \times x^2 + \cdots + nx^n$
$U_n = x + 2x^2 + \cdots + nx^n$
$U_{n+1} = x + 2x^2 + \cdots + nx^n + (n+1)x^{n+1}$
Donc $\boldsymbol{U_{n+1} = U_n + (n+1)x^{n+1}}$

Un peu plus…

$v_n = 1 + x + 1 + x^2 + \cdots + 1 + x^n$
$v_n = n + x + x^2 + \cdots + x^n$
$v_{n+1} = n + 1 + x + \cdots + x^n + x^{n+1}$
Donc $\boldsymbol{v_{n+1} = v_n + 1 + x^{n+1}}$

$C_{n+1} = 1 + x + \cdots + nx^n$
$+(n+1)x^{n+1}$
$\boldsymbol{C_{n+1} = C_n + (n+1)x^{n+1}}$

2) $A_{n+1} = 1 + 3 + \cdots + 3^n + 3^{n+1}$
$\boldsymbol{A_{n+1} = A_n + 3^{n+1}}$

$B_{n+1} = 2 + 4 + \cdots + 2(n+1)$
$\boldsymbol{B_{n+1} = B_n + 2n + 2}$

Corrigé Exercice 4

1) $\boldsymbol{u_1 = 13}$ et $\boldsymbol{r = 5} \Rightarrow$ $\boldsymbol{u_n = 13 + 5(n-1) = 8 + 5n}$ donc $\boldsymbol{u_{10} = 58}$
De $\boldsymbol{u_1}$ à $\boldsymbol{u_{10}}$ il y a 10 termes :

$\boldsymbol{S = 10 \times \frac{u_1 + u_{10}}{2} = 10 \times \frac{13+58}{2} = 5 \times 71 = 355}$

2) De 0 à 40 il y a 41 termes : $S = 930 \times \frac{1-0{,}6^{41}}{1-0{,}6} \simeq 2325$

3) De 1 à 25 il y a 25 termes : $w_1 + \cdots + w_{25} = 30 \times \frac{1-1{,}01^{25}}{1-1{,}01} \simeq 847$

4) $x_0 = 2500$ et $r = -20 \Rightarrow x_n = 2\,500 - 20n$ donc $x_{39} = 1720$
La somme des 40 premiers termes est :
$x_0 + \cdots + x_{39} = 40 \times \frac{2500+1720}{2} = 84\,400$

Un peu plus…

1) De 1 à 16 il y a 16 termes :
$S = 13 \times \frac{1-1{,}2^{16}}{-0{,}2} \simeq 1137$

2) $b_0 = -50$ et $r = -7$
On a $b_n = -50 - 7n$
Donc $b_{10} = -120$
et $b_{60} = -470$.
De 10 à 60, il y a 51 termes :
$S = 51 \times \frac{-120-470}{2} = -15\,045$.